INNOVATION, IMITATION, AND INTELLECTUAL PROPERTY RIGHTS

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The debate between the North and the South about the enforcement of intellectual property rights is examined within a dynamic general equilibrium framework in which the North invents new products and the South imitates them. A welfare evaluation of a policy of tighter intellectual property rights is provided by decomposing the welfare change into four items: (a) terms of trade; (b) production composition; (c) available products; and (d) intertemporal allocation of consumption. The paper provides a theoretical evaluation of the effect of each one of these items and their relative size. The analysis proceeds in stages. It begins with an exogenous rate of innovation in order to focus on the first two elements. The following two components are added by endogenizing the rate of innovation. Finally, the paper considers the role of foreign direct investment.

KEYWORDS: Innovation, imitation, intellectual property rights, international trade, foreign direct investment.

1. INTRODUCTION

Companies that engage in the development of new technologies, be it for the manufacturing of new goods, improved performance of existing products, or cheaper production processes, face difficulties in appropriating the fruits of their labor. Competing manufacturers attempt to imitate successful innovations and to adapt them to their own use. In the developed market economies patent, trademark, and copyright laws prevent the abuse of intellectual property rights. But even in those countries legal protection is imperfect, imitation is widespread, and often important information leaks out already during the development process (see Mansfield, Schwartz, and Wagner (1981) and Mansfield (1985)). This problem has been exacerbated by developing countries, some of which have not signed international treaties concerning protection of intellectual property rights and others that have laxly enforced domestic laws and regulations designed for this purpose (see Benko (1987)). A number of countries (including South Korea, Mexico, and Malaysia) have, however, initiated legislation designed to improve their record on this score.

Less developed countries often have a cost advantage in manufacturing, particularly due to low wages. For this reason their access to technologies enables them to compete effectively in product markets. This ability is mostly limited to industries that do not require highly skilled labor. Nevertheless, it has become a problem of major concern, as reflected, for example, in the fact that Trade Related Intellectual Property Rights have been put on the agenda of the

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recent Uruguay Round of trade negotiations under the GATT. One major reason for this development has been the relatively fast growth during the 1980s of trade in knowledge intensive products. By this I mean patented products, goods that are marketed under trademarks, as well as products with copyright protection, such as pharmaceuticals and videos. Trade in these products has grown much faster than overall trade in manufactures (see Maskus (1990)).

There exists very little evidence on the welfare effects of international infringements of intellectual property rights (IPRs). The U.S. Chamber of Commerce argued in the mid 1980’s that U.S. companies lose profits of about $60 billion per annum as a result of such infringements. This figure was considered to be grossly exaggerated. And indeed in a study conducted by the U.S. International Trade Commission (1988) that asked companies to evaluate their forgone profits on account of foreign infringements of IPRs, the figure was scaled down to about $24 billion for 1986. This too may be too high.

A study by Feinberg and Rousslang (1990) derived a decomposition of welfare losses from infringements of intellectual property rights by modeling the foreign market as consisting of a dominant firm and a competitive fringe. Using 1986 data they concluded that U.S. firms lost $2.3 billions in profits. They also computed aggregate gains of consumer surplus of about $3 billion for foreign and U.S. consumers. Foreign firms gained $0.6 billion. This suggests a net gain to the world economy of about $1.3 billion. The study employed strong assumptions. Refinements of these assumptions reduce significantly the net gains from looser intellectual property rights (see Maskus and Konan (1991)). Moreover, the study builds on a static framework which captures at best the effects of IPRs on existing products, while the effect of IPRs on the rate of innovation has been at the center of the debate.

A major argument in favor of tighter IPRs has been that they encourage innovation from which all the regions of the world benefit. A number of countries do not find this argument convincing, however. The counter argument has been that tighter intellectual property rights only strengthen the monopoly power of large companies that are based in industrial countries, to the detriment of the less developed countries. Brazil and India in particular have voiced concern over this matter in the recent round of trade negotiations.

Not only do there exist very few empirical studies that assess the importance of intellectual property rights in the debate between the developed and less developed countries, there also do not exist enough theoretical studies that provide a suitable framework for such an evaluation. All the studies that I could find employ a static partial equilibrium framework (see Chin and Grossman (1990), Diwan and Rodrik (1991), and Deardorff (1992)). To be sure, they do provide valuable insights. But it appears that the problem at hand is dynamic in nature, and therefore a dynamic analysis is called for.

I develop in this paper a number of dynamic general equilibrium models of two regions, North and South, which embody important elements that are pertinent to concerns about intellectual property rights. Innovation takes place in the North while the South imitates technologies that have been invented in
the North. The models build on the theory of endogenous growth and international trade that has been recently developed by Grossman and Helpman (1991b). This theory seems to be particularly suitable for the problem at hand. I identify four channels through which intellectual property rights affect these regions: (a) terms of trade; (b) interregional allocation of manufacturing; (c) product availability; and (d) R&D investment patterns. I provide welfare evaluations of each one of these effects and compare their relative size whenever they yield conflicting outcomes. Although this type of analysis is required for many policy issues, it is very rare for models of endogenous growth, and in particular for multi-country models.

Section 2 focuses on the first two effects in a simplified framework with an exogenous rate of innovation. Subsequently, in Section 3, I examine all four effects in an expanded framework that allows for endogenous rates of innovation. These two sections abstract from foreign direct investment. In principle the existence of FDI can modify in important ways the effects of IPRs on North and South. But the empirical evidence shows no relation between IPRs and foreign direct investment (see Maskus and Konan (1991), who also identify a significant relationship between IPRs and trade volumes). Nevertheless, I examine foreign direct investment in Section 4. Conclusions are provided in the closing section.

2. TWO BASIC CONSIDERATIONS

As I have pointed out in the introduction, supporters of tighter intellectual property rights (IPRs) in less developed countries employ the argument that lax IPRs reduce the innovative effort in developed countries and thereby hurt all countries that participate in the world’s trading system. I will examine in the next section the contribution of a responsive supply of R&D to the desirability of tighter IPRs. In this section, however, I employ a model with a constant rate of innovation in order to isolate two effects that play an important role independently of whether innovation is responsive to incentives: the interregional allocation of manufacturing and the terms of trade.

Tighter IPRs shift product lines from the less developed to the more developed region. As a result, demand for factors of production declines in less developed countries (LDCs) and increases in developed countries, thereby improving the developed countries’ terms of trade and worsening the terms of trade of the less developed countries. On this account the developed countries are better off with tighter IPRs while LDCs are worse off. On the other hand, efficiency calls for an allocation of manufacturing to the region with lower prices. Given lower wages in the less developed region, tighter IPRs bring about a shift of production to the higher price region, thereby reducing efficiency. On this account both regions lose.² It follows that unless there exist additional

² There exists an additional channel of influence that is not picked up by my model. Looser intellectual property rights may spur competition and bring about a decline in markups. This would lead to an overall welfare gain. My models have, however, built in constant markups.
considerations (to be discussed later on) the LDCs necessarily lose from tighter IPRs. Developed countries may gain or lose, however, depending on whether their gain from improved terms of trade outweighs their loss from the worsening of the interregional allocation of production. In the latter case both regions lose.

In what follows I construct a model in which only these two considerations exist. My analysis reveals structural features that determine the net welfare effects of tighter intellectual property rights. A central result is that in the presence of slow imitation a tightening of IPRs hurts the less developed as well as the developed region. When the rate of imitation is high, however, a tightening of intellectual property rights hurts the less developed region but benefits the developed region. It follows that there exists a conflict of interest about the desired policy change only when imitation is high to begin with.

In this section I use a simple model, based on Krugman (1979). There exist two regions, the industrial North and the less developed South. The North introduces new products at a constant exogenous rate \( g = \dot{n}/n \), where \( n \) equals the number (measure) of products society knows how to produce. The South imitates Northern products at the exogenous rate \( m = \dot{n}^S/n^N \), where \( n^S \) represents the number of products that the South knows how to produce while \( n^N \) represents the number of products that the South has not yet imitated; \( n^S + n^N = n \). The rate of imitation represents the fraction of products being imitated per unit time.\(^3\)

\[^3\] There is some abuse of terminology in this two-region division, because the less developed countries are not homogeneous even for the purpose at hand. Most technological imitation of the type considered in this paper takes place by newly industrialized countries (first and second generation), while the majority of LDCs engage in this activity only marginally (I thank Alessandro Golio for making this point; see his review in UNCTAD (1991a)). The former are, of course, the relevant group for our discussion.

\[^4\] The rate of innovation can also be viewed as endogenously determined in the following way. Suppose human capital is needed for innovation and it has no other use. Let the flow of new products per unit time be given by \( \dot{n} = K_a h/a_nH \), where \( K_a \) represents the stock of available knowledge in innovation (external to the individual innovator), \( h \) the employment of human capital in innovation, and \( a_nH \) a constant. Next suppose that the North has a fixed stock of human capital \( H \) and that the stock of knowledge equals cumulative experience in innovation, represented by \( n \). Then assuming full employment yields the constant equilibrium rate of innovation \( g = H/a_nH \).

\[^5\] In this formulation neither innovation nor imitation use up resources and the South is identified with the region that can imitate technologies but cannot innovate. This is a convenient simplification. A more complete model would specify innovation and imitation technologies that use up resources and derive endogenously which region innovates, which imitates, and at what rates. An extended model of this type would be much too complicated for the purpose at hand, which is to focus on welfare results. Some readers may be bothered by the specification \( \dot{n}^S = mn^N \), which implies that the flow of imitated products is proportional to the measure of products that have not been imitated so far but does not depend on the measure of products that have been imitated. This too is a convenient simplification. It has the desirable property that no imitation takes place when there are no products to be imitated and it captures the notion that the more products are available for imitation (\( n^N \)) the easier it is to imitate. One could also argue that the more products have been imitated the easier it is to imitate. In order to capture both ideas at once one can employ the specification \( \dot{n}^S = \mu \varphi(n^S, n^N) \), where \( \varphi(\cdot) \) is an increasing, linear homogeneous function, and \( \mu \) is a shift parameter. In this case the rate of imitation equals \( m = \dot{n}^S/n^N = \mu \varphi(n^S/n^N, 1) \) and a tightening of IPRs can be related to \( \mu \). One could proceed with the analysis using this specification, except that it would be more complicated.
Under these circumstances the number of products available at time $t$ equals

$$n(t) = n(0)e^{rt},$$

while the fraction of goods that have not been imitated, $\zeta = n^N/n$, obeys the differential equation

$$\dot{\zeta} = g - (g + m)\zeta.$$

The solution is

$$\zeta(t) = \bar{\zeta} + \left[\zeta(0) - \bar{\zeta}\right]e^{-(g + m)t}, \quad \bar{\zeta} = g/(g + m).$$

The fraction of goods that have not been imitated is a state variable, determined by past innovation and imitation. Its long-run steady state value equals $\bar{\zeta} = g/(g + m)$. This fraction attains a higher long-run value the faster the pace of innovation and the slower the rate of imitation.

We may interpret in this model a tightening of intellectual property rights as a decline in the rate of imitation; the stronger legal and administrative actions taken by the Southern government to protect Northern IPRs, the slower the pace of imitation. This view is, of course, incomplete, because it does not specify the mechanism through which government policies affect the rate of imitation.\(^7\)

It proves to be useful nevertheless in an attempt to clarify a number of important issues. Be it as it may, a change in $m$ affects the entire trajectory of $\zeta$. Let $m = \bar{m} - \mu$, where the initial value of $\mu$ equals zero. Then a tightening of intellectual property rights can be represented by an increase in $\mu$. From (3) we calculate (given the initial value of $\zeta$):

$$\frac{d\zeta(t)}{d\mu} = \left[1 - e^{-(g + m)t}\right]\frac{d\bar{\zeta}}{d\mu} + \left[\zeta(0) - \bar{\zeta}\right]t e^{-(g + m)t}, \quad \frac{d\bar{\zeta}}{d\mu} > 0.$$

When the economy begins in steady state, $\zeta(0) = \bar{\zeta}$, and the second term on the right-hand side equals zero. In this case the fraction of goods that have not been imitated increases at each point in time in response to a tightening of IPRs (except at $t = 0$). Since the total number of goods available does not change (see (1)), it follows that the North manufactures more goods and the South manufactures fewer goods at each point in time. In order to find out how this reallocation of production affects welfare, we need to specify the structure of production and preferences, to which I turn next.

Individuals have identical preferences in both regions. Their welfare equals the discounted flow of utility

$$U(t) = \int_t^{\infty} e^{-\rho(\tau-t)} \log u(\tau) \, d\tau,$$

where $\rho$ represents the subjective discount rate and $\log u(\tau)$ the flow of utility.

\(^6\) By definition, $\dot{\zeta}/\zeta = n^N/N - n/n$ and $n^N = \hat{n} - \hat{n}^\xi$. Together with the definitions of $g$ and $m$ we obtain (2).

\(^7\) See Grossman and Helpman (1991a) for a model with endogenous rates of innovation and imitation that are affected by government policies.
at time $\tau$. The flow of utility depends on consumption of the above mentioned $n$ products via a CES preference structure

$$u = \left[ \int_0^n x(j)^\alpha \, dj \right]^{1/\alpha}, \quad 0 < \alpha < 1,$$

where $x(j)$ represents consumption of product $j$. A product can be consumed only if supplied, and it can be supplied only if it has been invented. Consumers allocate spending only across the available products.

These preferences are homothetic. Therefore aggregate demand can be derived directly from individual preferences. As is well known the resulting demand functions have constant elasticities;

$$x(j) = p(j)^{-\varepsilon} \frac{E}{P^{1-\varepsilon}}, \quad \varepsilon = \frac{1}{1-\alpha} > 1,$$

where $E$ represents aggregate spending on consumer goods, $\varepsilon$ the elasticity of demand, and $P$ a price index such that

$$P = \left[ \int_0^n p(j)^{-\varepsilon} \, dj \right]^{1/(1-\varepsilon)}.$$

Substituting the demand functions (7) into (6), and taking account of (8), we obtain the indirect utility function

$$\log u = \log E - \log P.$$

Namely, the flow of utility equals the logarithm of real spending.

Next suppose that goods are manufactured with one unit of labor per unit output in both regions. A Northern manufacturer that invents a product can charge a monopoly price as long as his product has not been imitated. For the time being also assume that Northern inventors manufacture their products at home; i.e., they do not form a multinational corporation (I deal with multinationals in Section 4). In this case the demand functions (7) imply that the monopoly price of every product that has not been imitated equals

$$p^N = \frac{1}{\alpha} w^N,$$

where $w^N$ represents the wage rate in the North (that equals the manufacturer's marginal cost). It follows that the $n^N$ products that have not been imitated are

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8 An alternative interpretation would be to view the $n$ products as intermediate inputs and $u$ as a quantity of a homogeneous consumption good.

9 Imitation cannot emanate from Northern companies as long as price competition prevails (as I assume below) and there are positive private costs of imitation (no matter how small). This follows from the fact that in this case the imitator ends up with zero operating profits, because all Northern firms face the same constant unit manufacturing costs (see Grossman and Helpman (1991a)).
priced with a markup above Northern wages. I assume that the technology of a product that has been imitated by the South becomes available to all Southern manufacturers.\footnote{An alternative specification can also be used. Suppose that the technology of a product that has been imitated becomes available only to the imitator. Then he forms a duopoly with the Northern innovator. Assuming price competition, the Southern imitator (who has a cost advantage by assumption) takes over the entire market for the product. He engages in monopoly pricing if the gap between the Northern and Southern wage rates is large enough, or in limit pricing if it is not large enough (see Grossman and Helpman (1991a)). In the former case our results remain essentially intact.} In that event, competition leads to marginal cost pricing of the remaining \( n^S \) products;

\begin{equation}
\tag{11}
 p^S = w^S,
\end{equation}

where \( w^S \) stands for the wage rate in the South. We assume that the wage rate is higher in the North. Therefore the price of goods manufactured in the North is also higher.

Taking account of the pricing practices (10) and (11), the price index \( P \) can be represented by (see (8)):

\begin{equation}
\tag{12}
P = n^{1/1-\varepsilon} \left[ \zeta (p^N)^{1-\varepsilon} + (1 - \zeta) (p^S)^{1-\varepsilon} \right]^{1/1-\varepsilon}.
\end{equation}

Since welfare is higher the higher is real spending, an increase in the fraction of goods manufactured in the North hurts every consumer with a given nominal expenditure level. This is because—other things equal—an increase in \( \zeta \) raises the price index and reduces real spending. Other things do not remain equal, however, when the fraction of goods manufactured in the developed region rises as a result of tighter intellectual property rights. In order to see what additional changes take place, we need to consider the labor markets.

Denote by \( x^N \) output per product in the North and by \( x^S \) output per product in the South. In a similar vein denote by \( L^i \) the labor force of region \( i, i = N, S \). Then we can express the labor market clearing conditions by

\begin{equation}
\tag{13}
n^i x^i = L^i, \quad i = N, S.
\end{equation}

From the demand functions (7) and the labor market clearing conditions (13) we obtain

\begin{equation}
\tag{14}
\frac{p^N}{p^S} = \left[ \frac{L^S}{L^N} \frac{\zeta}{1 - \xi} \right]^{1/\varepsilon}.
\end{equation}

This shows that shifts in the interregional distribution of manufacturing affect relative prices; the terms of trade improve in the North and deteriorate in the South as a result of tighter intellectual property rights (because the fraction of goods manufactured in the North increases; see the discussion following equation (4)).
We are now in a position to consolidate the welfare effects of tighter IPRs. The formal model has identified two channels of influence on regional welfare levels: (a) the interregional allocation of production; and (b) terms of trade movements. It is quite clear that both lead to a welfare loss for the South. The remaining question is whether they necessarily bring about a welfare gain for the North. In order to answer this question we need to perform some calculations.

Income per capita in the South equals \( p^S \) while income per capita in the North equals \( p^N \), despite the fact that the North derives income from both labor services and profits while the South derives income only from labor services.\(^{11}\) All income is equally distributed across workers within a region and trade is balanced. In this event consumer spending per worker in region \( i \) equals \( p^i \), \( i = N, S \). It follows from (9) and (12) that the flows of utility of typical workers in the North and South equal

\[
(15) \quad \log u^N = \frac{1}{\varepsilon - 1} \log n + \frac{1}{\varepsilon - 1} \log \left[ \xi + (1 - \xi) \left( \frac{p^N}{p^S} \right)^{\varepsilon^{-1}} \right],
\]

\[
(16) \quad \log u^S = \frac{1}{\varepsilon - 1} \log n + \frac{1}{\varepsilon - 1} \log \left[ \xi \left( \frac{p^S}{p^N} \right)^{\varepsilon^{-1}} + (1 - \xi) \right],
\]

where use has been made of the fact that spending on consumer goods \( E \) equals \( p^N \) for a Northern worker and \( p^S \) for a Southern worker. Since Northern manufacturers charge a higher price than their Southern competitors, it is evident from these equations that for given terms of trade \( p^N/p^S \) an increase in the share of goods manufactured in the North reduces welfare of typical workers in both regions. In addition, the induced rise in the relative price of Northern products (see (14)) benefits Northern workers and hurts Southern workers (recall that \( \varepsilon > 1 \)).

In order to calculate the overall effect of a change in the intellectual property rights regime on a Northern worker’s welfare we use the intertemporal preference structure (5) to obtain

\[
(17) \quad \frac{dU^N(0)}{d\mu} = \int_0^\infty e^{-\rho t} \frac{d \log u^N(t)}{d\mu} \, dt.
\]

For economies that are initially in steady state (i.e., \( \xi(0) = \bar{\xi} \)), equation (17)

\(^{11}\) In the South all income derives from labor services and aggregate income equals \( w^S L^S \). Therefore income per worker (assuming that \( L^S \) represents the number of workers) equals the wage rate. But the wage rate equals the price of a Southern product (see (11)). The North derives income from both labor and profits, and aggregate income equals \( n^N p^N + w^N L^N \), where \( p^N \) represents profits per product. From the pricing equation (10) profits per product equal the fraction \( (1 - \alpha) \) of revenue, or \( \pi^N = (1 - \alpha) p^N x^N \). Taking account of the labor market clearing condition (13) and the pricing equation (10) this implies that aggregate income equals \( p^N L^N \). Therefore income per worker equals the price of a Northern product.
together with (4), (14), and (15) imply

\[ \frac{dU^N(0)}{d\mu} = \frac{1}{K} \left\{ 1 - \left( 1 - \frac{\alpha}{\bar{\xi}} \right) \left( \frac{L^S}{L^N} \frac{\bar{\xi}}{1 - \bar{\xi}} \right)^{\alpha} \right\}, \]

where

\[ K = (e - 1) \rho (\rho + g + m)(g + m) g^{-1} \left[ \frac{\bar{\xi} + (1 - \bar{\xi})^{1-\alpha}}{\bar{\xi}} \bar{\xi}^\alpha \left( \frac{L^S}{L^N} \right)^\alpha \right] > 0. \]

The term in the second parenthesis on the right hand side of (18) is larger than one as long as Northern products command higher prices (see (14)). The term \( \alpha/\bar{\xi} \) can be smaller or larger than one (recall that \( \bar{\xi} = g/(g + m) \)). It follows that the term in braces on the right hand side can be positive or negative. If \( \alpha/\bar{\xi} \) exceeds one, for example, Northern workers gain from a tightening of intellectual property rights. In this case their gain from improved terms of trade outweighs their loss from the interregional reallocation of production. When \( \alpha/\bar{\xi} \) falls short of one, however, they may gain or lose. It is instructive to see how these possibilities depend on the initial level of the rate of imitation.

First consider a world in which the rate of imitation is very small (close to zero). In this case the share of goods manufactured by the North is close to one and the right hand side of (18) is negative (the price of goods then is necessarily lower in the less developed region). It follows that a tightening of intellectual property rights harms both the South and the North. Another way to put it is that a small degree of imitation benefits both regions. As the rate of imitation increases, however, the right hand side of (18) also increases and it attains the value zero for some critical rate of imitation \( m_c \) (holding constant all other parameters). The right hand side is positive for all rates of imitations that exceed this critical level. Naturally, the range of feasible rates of imitation is determined by the requirement that wages be lower in the South, or \( m \in (0, \alpha^* g L^S / L^N) \) (see (10), (11), and (14)). Therefore if \( m_c \in (0, \alpha^* g L^S / L^N) \), there exists a range of feasible imitation rates \( m \in (m_c, \alpha^* g L^S / L^N) \) for which tightening of IPRs benefits the North and hurts the South. We have thus established the following proposition.

**Proposition 1:** For economies in steady state there always exists a range of feasible imitation rates \( 0 < m < \min (m_c, \alpha^* g L^S / L^N) \) in which a tightening of intellectual property rights hurts both regions. If \( m_c < \alpha^* g L^S / L^N \), however, there also exists a range of feasible imitation rates \( m_c < m < \alpha^* g L^S / L^N \) in which a tightening of intellectual property rights benefits the North and hurts the South.

This proposition establishes conditions under which the tightening of intellectual property rights benefits or hurts the North. As is evident from the proposition, the range of imitation rates in which the North loses expands as the critical value \( m_c \) increases. An examination of the right hand side of (18) reveals that \( m_c \) rises with the relative size of the South \( L^S / L^N \) and with the rate of innovation \( g \). Therefore, the larger the relative size of the South or the rate of
innovation, the larger the range of imitation rates in which the North suffers from tighter intellectual property rights. The North necessarily loses from tighter IPRs whenever the rate of imitation is small.

An intuitive explanation of Proposition 1 can be given as follows. A low rate of imitation gives rise to a large steady state wage differential between the North and the South. In the limit, as the rate of imitation shrinks to zero, the wage differential becomes unbounded. Then production in the South is negligible and the North is virtually a closed economy. Starting from these initial conditions a slight increase in the rate of imitation brings about an expansion of output in the South (i.e., a reallocation of production) and an expansion of trade between the North and the South. As a result, the North can exploit its monopoly power (which it could not to begin with) and also enjoy some gains from trade. As the deterioration of the terms of trade has only a negligible effect under these circumstances, these benefits dominate the welfare calculus.\(^ {12}\)

This intuition suggests that the initially large wage differential plays a major role in ensuring that the welfare level of a Northern resident rises as the rate of imitation increases from zero to some positive level. For this reason we can examine the robustness of this result by considering a model that limits equilibrium wage differentials. To this end consider a model in which in addition to the differentiated products every region can manufacture an outside good with one unit of labor per unit output. It can be shown that in this case Northern welfare does not necessarily increase when the rate of imitation rises from zero to a small positive level. On the other hand, when the outside good is only slightly desirable there typically exists close to the origin a range of imitation rates in which the North loses from tighter intellectual property rights.\(^ {13}\)

3. ENDOGENOUS INNOVATION

Our discussion so far has neglected the effects of tighter intellectual property rights on the rate of innovation. As pointed out in the introduction this issue often is raised in public policy discussions. I examine in this section the response of the rate of innovation to tighter IPRs in the short run as well as the long run, and the overall implications of this response for the welfare of the South and the North. An important result of this inquiry is that the rate of innovation rises initially and subsequently declines. The temporary acceleration of innovation

\(^ {12}\) It should be noted that in this framework the reallocation of production from North to South does not change aggregate profits of Northern manufacturers. This results from the fact that profit making is constrained by the availability of labor in the North and by the markup, and both are constant. We will see in Section 4 how this changes in the presence of multinational corporations.

\(^ {13}\) The preceding discussion is based on an analysis suggested by Martin Hellwig, in which the utility function \(\log u\) is replaced with \(\log \tilde{u} = \log u + \delta \log c\), where \(c\) is the consumption of the outside good and \(\delta\) is a nonnegative parameter (my specification is a special case in which \(\delta\) equals zero). This model yields two interesting results: (a) the welfare level of a Northern resident increases with a tightening of IPRs whenever \(\delta > 1/(\epsilon - 1)\); and (b) for \(\delta\) close enough to zero the welfare level of a Northern resident is lower when the rate of imitation equals zero than when it is positive but small.
not withstanding, the overall welfare effect of the shift in the time pattern of available products proves to be detrimental to both regions. Adding these effects to the welfare contribution of the terms of trade movements and the interregional allocation of production (that were discussed in the previous section), as well as the intertemporal shifts in the R&D investment rate, we can calculate overall welfare changes. Although these prove to be ambiguous, the calculations identify structural features that determine the bottom line of this welfare calculus. An important result that emerges is that despite the endogenous response of the rate of innovation, both regions lose from tighter IPRs whenever imitation proceeds at a slow pace.

It proves most convenient to begin this discussion with the help of a simple approach developed by Grossman and Helpman (1991a), in which innovation requires labor input.\(^{14}\) Let the invention of new products per unit time \(\dot{n}\) equal \(lK_n/a\), where \(a\) represents a productivity parameter in innovation, \(l\) is labor employed in R&D, and \(K_n\) is the cumulative stock of knowledge in the inventive activity. The latter is taken to equal \(n\). Therefore \(g = l/a\), and the resource constraint for the North becomes

\[
ag + n^{N} \times N = L^{N}
\]

rather than (13) (the latter remains valid for \(i = S\)). The first term on the left-hand side represents employment in R&D while the second term represents employment in manufacturing.

If investment in R&D takes place, the reward from this activity, which consists of a blueprint whose value we denote by \(v^N\), must cover R&D outlays. The equilibrium value of a blueprint cannot exceed these outlays, because otherwise the demand for labor by innovators becomes unbounded (we assume free entry into innovation). If the value of a blueprint falls short of R&D costs, however, no rational agent invests in R&D. It follows that in an equilibrium with positive innovation R&D costs just equal the value of a blueprint. Given the above specified R&D technology, this condition becomes

\[
v^N = w^N a / n.
\]

Now assume that the developed region has well functioning financial markets, including a stock market. Then the value of a firm (which we associate with a blueprint) equals the present value of its expected stream of profits, because firms face idiosyncratic risks that can be diversified away via portfolio holdings. In fact, the only risk faced by a firm is that its product will be imitated by the South. In a time interval of length \(dt\) the South imitates a proportion \(mdt\) of northern products. Let the firms targeted for imitation be chosen randomly with a uniform distribution. Then every product that has not previously been imitated faces the probability \(mdt\) of being imitated in the next time interval of

\(^{14}\) They also assume that imitation requires labor input. For present purposes, however, I maintain the assumption that the rate of imitation depends only on the degree of enforcement of intellectual property rights.
length $dt$ (put differently, $m$ represents the hazard rate of being imitated). Under these circumstances arbitrage in asset markets implies (see Grossman and Helpman (1991a)):

$$\frac{\pi^N}{v^N} + \frac{\dot{v}^N}{v^N} = r^N + m,$$

where $\pi^N$ represents profits per unit time and $r^N$ stands for the nominal interest rate in the North. This is a standard no arbitrage condition in asset markets. It states that the inverse of the price earning ratio (the first term on the left-hand side) plus the rate of capital gain on equity holdings equals the risk adjusted interest rate. Here, risk adjustment relates to the instantaneous probability of being imitated.\(^{15}\)

Northern products are priced according to (10). Therefore, profits per product account for a fraction $(1 - \alpha)$ of revenue; i.e., $\pi^N = (1 - \alpha)p^N x^N$. Using the resource constraint (19) it follows that $\pi^N = (1 - \alpha)p^N(L^N - a_\zeta)/n^N$. Substituting this equation together with (10) and (20) into (21) implies

$$\frac{1 - \alpha}{\alpha} \frac{L^N - a_\zeta}{a_\zeta} + \frac{\dot{v}^N}{v^N} = r^N + m.$$

It remains to derive an expression for the interest rate.

As is well known, a Northern consumer who maximizes welfare with preferences (5) and an indirect utility function (9) subject to an intertemporal budget constraint chooses a rate of growth of consumption spending that matches the difference between the interest rate and his subjective rate of time preference; i.e.,

$$\frac{E^N}{E^N} = r^N - \rho.$$

Now assume that no financial capital flows between the two regions, so that the North finances investment in R&D entirely with domestic savings. The lack of international mobility implies that the trade account is balanced at every point in time, or $E^N = p^N n^N x^N$. Using this relationship together with (10), (19),

\(^{15}\) This equation can be derived directly from the observation that the value of a firm equals the expected present value of its profits stream by differentiating this equality. Alternatively one can use the following no arbitrage argument. In a time interval of length $dt$ the firm provides a profit stream of $\pi^N dt$. In addition it provides a capital gain of $\dot{v}^N dt$ if not imitated and a capital loss of $v^N dt$ if imitated (the latter results from the fact that profits drop to zero for products that have been imitated; therefore imitation leads to a total capital loss). The probability of being imitated in this time interval equals $mdt$ and the probability of not being imitated equals $(1 - mdt)$. Therefore the expected reward for the owners of the firm equals $\pi^N dt + (1 - mdt)\dot{v}^N dt - mdt$. If they were to sell their shares and make a consumption loan they would receive interest income of $r^N v^N dt$ during the same time interval. Arbitrage in asset markets implies that these two forms of asset holdings yield the same reward. Dividing this equality by $dt$ and taking the limit as $dt$ approaches zero implies (21).
(20), (22), and (23), we obtain a differential equation for the rate of innovation:\(^\text{16}\)

\[
\dot{g} = \left( \frac{L^n}{a} - g \right) \left[ \rho + m + g - \frac{1 - \alpha}{\alpha} \left( \frac{L^n}{a} - g \right) \frac{1}{\zeta} \right].
\]

Equation (24) together with the differential equation (2) for the fraction of goods that have not been imitated, which I reproduce here for convenience,

\[
\dot{\zeta} = g - (g + m) \zeta,
\]
form an autonomous system of two differential equations in \((g, \zeta)\). In this system, \(\zeta\) is a state variable while \(g\) is a jump variable.

Figure 1 depicts the phase diagram for this system. The shaded area represents a nonfeasible region in which either the share of goods not yet imitated exceeds one or R\&D employment exceeds the labor force in the North. The

\(^{16}\) From the relationship between spending and the value of production, \(E^n = p^n N_n x^n,\) and the resource constraint (19), we obtain

\[
\frac{\dot{E}^n}{E^n} = - \frac{ag}{L^n - ag} + \frac{\dot{p}^n}{p^n}.
\]

From the valuation of firms on the supply side, however, given by (20), and the pricing of goods (10), the rate of increase in the price of Northern products equals the rate of appreciation of a firm plus the rate of innovation. Substituting this relationship together with the previous equation into (23), we obtain an expression for the interest rate

\[
r^n = \rho + g - \frac{ag}{L^n - ag} + \frac{\dot{u}^n}{u^n}.
\]

Substituting this result into the no arbitrage condition (22) yields the differential equation (24).
dotted area represents a nonfeasible region in which the wage rate in the North is lower than in the South, defined by\textsuperscript{17}

\begin{equation}
\frac{g}{a} < \frac{1}{1 - \xi} \left( L^N - \frac{\xi}{\alpha} \alpha^\ell L^S \right).
\end{equation}

The equation for the heavy-line curve \( \dot{\xi} = 0 \) can be represented by

\begin{equation}
\xi = \frac{1}{\alpha} \left( \frac{L^N}{a} - \frac{1}{\rho + m + g} \right),
\end{equation}

while the equation for the heavy-line curve \( \dot{\xi} = 0 \) can be represented by

\begin{equation}
\xi = \frac{g}{g + m}.
\end{equation}

The intersection of these two curves at point \( A \) describes the steady-state long-run equilibrium. All trajectories that do not converge to this point violate an equilibrium requirement over some time interval (see Grossman and Helpman (1991b)). The equilibrium trajectory consists of the saddle path that converges to \( A \). Along this saddle path the rate of innovation declines and the fraction of products that have not been imitated increases over time whenever the latter variable falls short of its steady-state value, while the rate of innovation increases and the fraction of products that have not been imitated declines over time whenever the latter variable exceeds its steady-state value.

As is evident, my discussion has been confined to an economic environment in which there exists long-run innovation in the North and the wage rate is lower in the South. This combination is, of course, of particular interest for the problem at hand. A natural question that arises, however, is whether this combination is likely to emerge; namely, whether point \( A \) is likely to fall outside the dotted area. The answer depends on structural parameters, and it again proves useful to examine its dependence on the rate of imitation. Observe that the dotted area does not depend on the rate of imitation, while the location of point \( A \) does depend on \( m \). In particular, it follows from (27) and (28) that a reduction in the rate of imitation shifts both curves \( \dot{g} = 0 \) and \( \dot{\xi} = 0 \) to the right, with the latter shifting by proportionately more. The result is that the long-run equilibrium point shifts down and to the right, implying that the long-run rate of innovation \( \bar{g} \) declines and the long-run fraction of products that have not been imitated \( \bar{\xi} \) increases. In the limit, as the rate of imitation \( m \) approaches zero, \( \bar{\xi} \) approaches one and the rate of innovation approaches \((1 - \alpha)(L^N/a) - \alpha \bar{\rho}\). This describes the rate of innovation attained by the North when it does not trade with the South, provided that this rate is positive. Otherwise the autarky rate of innovation in the North equals zero (see Grossman and Helpman\textsuperscript{17}).
Assume that an isolated North innovates at a positive rate; i.e., $(1 - \alpha)(L_N/a) > \alpha \rho$. Then for $m$ approaching zero the long-run equilibrium approaches a point on the vertical broken line passing through $\zeta = 1$. This point always lies outside the dotted area. We conclude that, given the model's parameters, there exists an upper bound on the imitation rate, $m_{\text{max}}$, such that point $A$ falls in the feasible region for all $m \in (0, m_{\text{max}})$. I confine the following analysis to $m \in (0, m_{\text{max}})$.

Our preceding analysis has established in passing that the long-run rate of innovation declines and the long-run fraction of unimitated products increases in response to tighter intellectual property rights.\(^{18}\) These results are important for what follows and deserve to be stated as a proposition:

**Proposition 2:** $d\tilde{g}/d\mu < 0$ and $d\tilde{\zeta}/d\mu > 0$.

The first part of this proposition shows that tighter IPRs need not stimulate innovation in the long run (see, however, the next proposition for short-run effects). In order to understand how this case arises, combine (27) with (28) into a single equation that provides an implicit solution for the long-run rate of innovation:

\[
(29) \quad \frac{1 - \alpha}{\alpha} \left( \frac{L_N}{a} - g \right) \frac{m + g}{g} = \rho + m + g.
\]

The left hand side represents the profit rate (i.e., inverse of the price earning ratio). The right hand side represents the effective cost of capital, inclusive of a risk premium. A tightening of intellectual property rights (a reduction of $m$) reduces the effective cost of capital as well as the profit rate. Moreover, its impact on the effective cost of capital is smaller in size than on the profit rate. The result is that it becomes less profitable to innovate, and the rate of innovation declines.\(^{19}\)

A proper welfare evaluation requires knowledge of the effects of tighter IPRs on the entire trajectory of $g$ and $\zeta$, and not only on their steady state values (see Diamond (1980) and Judd (1982)). We turn now to this calculation. I restrict the analysis to economies that are initially in steady state; namely, $\zeta(0) = \tilde{\zeta}$. For economies of this type we can calculate the first order response of

\(^{18}\) A related result appears in Grossman and Helpman (1991a).

\(^{19}\) The intuition behind this result will be discussed after the derivation of the short-run effect of a tightening of IPRs on the rate of innovation. It should, however, be understood that the long-run negative link between the rate of innovation and the rate of imitation is not robust. A somewhat different model of North-South trade with innovation and imitation has been developed in Grossman and Helpman (1991b, Ch. 12) and in that model the link can go either way. In fact, even in the model employed in this paper the link can go either way if instead of the logarithmic utility function one were to use a more general functional form with a constant elasticity of the marginal utility of consumption. With the more general utility function the left hand side of (29) remains the same while the right hand side becomes $\rho + m + (1 - (1 - \alpha)\sigma/(1 - \alpha))g$, where $\sigma$ represents the elasticity of the marginal utility of consumption. When $\sigma = 1$ we obtain (29). But with different values of $\sigma$ it is possible to generate a negative link between the rate of innovation and the rate of imitation. The key for the latter is to obtain a right hand side that rises with $m$ proportionately more than the left hand side. This happens, for example, when $\alpha = 1/2$ and $\rho + \sigma g < g$. Due to this lack of robustness the following results are indicative of what may happen rather than definitive.
(\(\zeta, g\)) to a tightening of intellectual property rights from a linearized version of the differential equations (24) and (25) around their steady state values. Taking into account the initial condition \(\zeta(0) = \tilde{\zeta}\), I show in the Appendix that

\[
\frac{d\zeta(t)}{d\mu} = (1 - e^{-\lambda t}) \frac{d\tilde{\zeta}}{d\mu},
\]

\[
\frac{dg(t)}{d\mu} = \frac{d\tilde{g}}{d\mu} + \Lambda e^{-\lambda t} \frac{d\tilde{\zeta}}{d\mu},
\]

where \(\lambda\) and \(\Lambda\) are positive. It follows from these equations and from Proposition 2 that the fraction of goods that have not been imitated rises at each point in time following a tightening of IPRs (except for \(t = 0\)). On the other hand, although the rate of innovation declines in the long run, (31) suggests that it may increase in the short run, and especially so if the long-run response of the rate of innovation is weak.\(^{20}\)

A larger share \(\zeta\) hurts the South on account of a worsening of its terms of trade and a worsening of the interregional allocation of production. The North is also hurt by the worsening of the interregional allocation of production, but unlike the South it gains from improved terms of trade. These points, which we discussed in the previous section, also apply in the current setup. Both regions would lose on account of slower growth in the world economy if the rate of innovation were to decline at each point in time.\(^{21}\) But if the rate of innovation were to rise temporarily, both regions might gain on account of a spurt in growth in the world economy, even though eventually the spurt is reversed.

Can the rate of innovation rise temporarily as a result of tighter IPRs? The answer to this question is that in fact it must. In order to develop some intuition for this result, consider the limiting case with \(\rho = 0.\)\(^{22}\) The steady state equations (27) and (28) imply that for \(\rho = 0\) a reduction of \(m\) shifts both curves \(\zeta = 0\) and \(g = 0\) in Figure 1 rightward by the same proportion. Consequently the long-run equilibrium point \(A\) shifts horizontally to \(B\) in Figure 2. The new equilibrium trajectory that passes through \(B\) slopes downward, as depicted. Therefore initially the system jumps from \(A\) to \(A'\) and subsequently follows the saddle path to \(B\). Evidently the rate of innovation remains higher at all points in time until it reaches its steady-state value. Now continuity implies that for \(\rho\) positive but close to zero a tightening of intellectual property rights increases the rate of innovation over a finite time interval.

\(^{20}\) This observation follows from the fact that the first term on the right hand side of (31) is negative while the second term is positive.

\(^{21}\) Because the first term on the right hand side of (15) and (16) is smaller in this case at each point in time.

\(^{22}\) The subjective discount rate cannot equal zero, because it would imply unbounded utility \(U(0)\). This utility level is, however, finite for all positive values of \(\rho\). For this reason we should think about this exercise as applying to the limit of a sequence of economies with \(\rho \to 0\).
Next I establish that the rate of innovation must increase on impact. From the steady-state equations (27)–(28) we calculate:

\[
\frac{d\tilde{g}}{d\mu} = \tilde{g} \frac{1}{D} \frac{1}{\alpha},
\]

\[
\frac{d\bar{g}}{d\mu} = -\frac{\bar{g}}{D} \rho,
\]

where

\[
D = m \rho + \frac{1}{\alpha} (m + \bar{g})^2.
\]

Together with (30) these expressions imply

\[
\frac{dg(0)}{d\mu} = \frac{\bar{g}}{D\alpha} (\Lambda - \alpha \rho).
\]

I show in the Appendix that

\[
\alpha \rho < \Lambda < \alpha (\rho + \lambda).
\]

It follows from (35) and (36) that the rate of innovation rises on impact. By continuity this also implies the following proposition.
**Proposition 3:** For economies in steady state there exists a time interval $[0, T)$ such that $dg(t)/d\mu > 0$ for $t \in [0, T)$.

We have established that a tightening of intellectual property rights initially raises the rate of innovation, but that the rate of innovation subsequently declines. The intuition behind this result is the following. A reduction in the rate of imitation increases the expected tenure of a monopolistic supplier in the market. If the rate of innovation does not change his risk adjusted cost of capital, $\rho + g + m$, declines. At the same time the profit rate $\pi^N/\nu^N = (1 - \alpha)(L^N/a - g)/\alpha \zeta$ does not change, because the fraction $\zeta$ does not change on impact. In that event the profitability of R&D rises, more resources get attracted to innovation, and $g$ increases on impact. The increase in $g$ raises the price-earning ratio and mitigates the decline in the effective cost of capital. As the pace of innovation rises, however, the difference between the profit rate and the effective cost of capital declines over time as a result of a gradual increase in the share of products that have not been imitated $\zeta$ (the latter reduces the profit rate). These developments lead to a gradual reduction in the rate of innovation until it declines in the long run.

Our results imply that the number of products available to consumers rises in the initial phase but declines in the long run. Since consumers value product choice (i.e., welfare rises with the number of available products; see (15) and (16)), their flow of utility rises in the initial phase on account of the endogenous response of the rate of innovation but eventually declines. In order to obtain the overall welfare effect of these changes we need to expand our calculations.

First observe that the South spends all of its income on consumer goods, because there exist no international capital flows and no investment takes place in the South. As a result, the utility flow of a Southern worker, whose income equals $p^S$ per unit time, remains the same as in (16). Consequently the increase in the present value of a Southern worker's utility flow, discounted with the subjective rate of time preference, can be calculated as

$$
\frac{dU^S(0)}{d\mu} = \frac{1}{e - 1} (\Delta_n + \Delta_\xi),
$$

where

$$
\Delta_n = \frac{d}{d\mu} \int_0^\infty e^{-\rho t} \log n(t) \, dt,
$$

$$
\Delta_\xi = \frac{d}{d\mu} \int_0^\infty e^{-\rho t} \log \left[ \zeta(t) \left( \frac{p^S(t)}{p^N(t)} \right)^{e-1} + 1 - \zeta(t) \right] \, dt,
$$

and relative prices equal (see footnote 17):

$$
\frac{p^N(t)}{p^S(t)} = \theta(t)^{1/e}, \quad \theta(t) = \frac{L^S}{L^N - ag(t)} \frac{\zeta(t)}{1 - \zeta(t)}.
$$
The term $\Delta_n$ captures the welfare effect of product availability while $\Delta^\xi$ captures the effect of real spending for a given profile of available products. The latter represents changes that emanate from the terms of trade and the interregional allocation of production. The South’s terms of trade improve if and only if $\theta$ declines.

The calculation for the North is somewhat more complicated, because the North saves and invests in R&D. As I have already pointed out, the lack of international capital mobility implies aggregate Northern spending on consumer goods $E^N = p^N n_N x_N$, which together with the resource constraint (19) implies $E^N = p^N (L^N - ag)$. Thus, spending per worker equals $p^N(1 - ag/L^N)$, where $ag/L^N$ represents the savings and investment rate. In the previous section, where there was no saving and no investment, spending per worker equaled $p^N$. Now we need to replace equation (15) which describes the flow of utility, with

$$\log u^N = \frac{1}{\epsilon - 1} \log n + \frac{1}{\epsilon - 1} \log \left[ \xi + (1 - \xi)(p^N/p^S)^{\epsilon - 1} \right]$$

$$+ \log \left( 1 - \frac{ag}{L^N} \right),$$

in order to account for savings. The last term on the right hand side represents the effect of savings on the current utility flow; the larger the savings rate the lower is current utility from consumption. From (41) we calculate the change in the discounted flow of utility of a Northern worker as

$$\frac{dU^N(0)}{d\mu} = \frac{1}{\epsilon - 1} (\Delta_n + \Delta^\xi) + \Delta^\delta,$$

where $\Delta_n$ is given in (38),

$$\Delta^\xi = \frac{d}{d\mu} \int_0^\infty e^{-\rho t} \log \left[ \left( 1 - \xi(t) \right) \frac{p^N(t)}{p^S(t)} \right]^{\epsilon - 1} + \xi(t) \right] dt,$$

$$\Delta^\delta = \frac{d}{d\mu} \int_0^\infty e^{-\rho t} \log \left[ 1 - \frac{ag(t)}{L^N} \right] dt.$$

Here $\Delta_n$ captures again the welfare effect of product availability, $\Delta^\xi$ represents the welfare effect of real income for a given time profile of available products, and $\Delta^\delta$ represents the welfare effect of the endogenous shifts in saving rates.

Using equations (31)–(33) and the fact that $\log n(t) = \log n(0) + \int_0^t g(\tau) d\tau$, we calculate from (38):

$$\Delta_n = \frac{\bar{g}}{\alpha \rho D(\rho + \lambda)} \left[ L - \alpha(\rho + \lambda) \right].$$

Together with (36) this equation proves the following:

**Proposition 4:** For economies that begin in steady state, a tightening of intellectual property rights hurts both regions via its effect on product availability; i.e., $\Delta_n < 0$. 

This result is quite striking. Since the rate of innovation rises initially and declines subsequently, one would expect product availability to raise welfare for high subjective discount rates and reduce welfare for low subjective discount rates. The reason that this intuition does not apply is that a higher subjective discount rate also leads to a larger long-run decline and a smaller short-run increase of the rate of innovation (see (31)–(33)).

In order to complete our welfare analysis for the South we need to consider the term \( \Delta_s^S \) that embodies changes in the interregional allocation of production and the terms of trade. It proves in fact useful to decompose this expression into

\[
\Delta_e^S = \Delta_x^S + \Delta_y^S,
\]

where the first term on the right hand side represents the effect on (39) of changes in \( \zeta(t) \) holding relative prices constant and the second term represents the effect on (39) of changes in relative prices holding constant the weights \( \zeta(t) \) and \( [1 - \zeta(t)] \) in equation (39). These two effects operate here very much in the same way as in the simpler model discussed in the previous section. The difference is that now changes in the rate of innovation also affect the terms of trade (see (40)). In particular, since the rate of innovation rises initially and declines subsequently, the reallocation of resources between R&D and manufacturing brings about an initial deterioration of the South’s terms of trade and a subsequent improvement (because an expansion of R&D raises the demand for Northern labor). I show, however, in the Appendix, that the endogenous response of the rate of innovation notwithstanding the overall welfare effect of the terms of trade is negative for the South; i.e., \( \Delta_y^S < 0 \). It is also straightforward to see from (39) that the production reallocation effect reduces the South’s welfare; i.e., \( \Delta_x^S < 0 \). Proposition 5 follows.

**Proposition 5:** For economies that begin in steady state, tighter intellectual property rights hurt the South on account of both changes in the interregional allocation of production and changes in the terms of trade; i.e., \( \Delta_x^S < 0 \) and \( \Delta_y^S < 0 \).

Propositions 4 and 5 imply that the South loses from tighter intellectual property rights through all of the applicable channels, which proves the following theorem.

**Theorem 1:** For economies that begin in steady state, the South loses from tighter intellectual property rights.

This theorem establishes the important result that the temporary acceleration of the rate of innovation in response to tighter intellectual property rights is never sufficient to compensate the South for its losses. And as we have seen, the South has no other sources of gain.
Next consider the welfare of the North. We have already seen in Proposition 4 that the North loses as a result of the shift in the time profile of available products. As (42) shows, however, we need to compare this loss to the changes in welfare that result from shifts in real income for a given profile of product availability and the adjustments in the savings rate. From (44) together with (31)–(33) I calculate the contribution of the savings rate, or the intertemporal reallocation of R&D expenditure, to be

\[
\Delta^N_s = -\frac{\bar{g}}{\alpha D(\rho + \lambda)} \left[ \Lambda - \alpha(\rho + \lambda) \right] \left( \frac{L^N}{a} - \bar{g} \right)^{-1}.
\]

This is positive in view of (36), demonstrating that adjustments of savings and R&D investment are welfare enhancing. A comparison of (46) with (45) shows, however, that \((1/(\varepsilon - 1)) \Delta^N_n + \Delta^N_s < 0\), which proves the following proposition.

**Proposition 6:** For economies that begin in steady state a tightening of intellectual property rights raises the North's welfare on account of adjustments in the savings and R&D investment rates, but this welfare gain is smaller than the welfare lost inflicted by the shift in the time profile of available products; i.e., \(\Delta^N_s > 0\) and \((1/(\varepsilon - 1)) \Delta^N_n + \Delta^N_s < 0\).

It remains to evaluate \(\Delta^N_e\). As in the discussion of the South we can decompose \(\Delta^N_e\) into a production allocation effect and a terms of trade effect:

\[
\Delta^N_e = \Delta^N_c + \Delta^N_\theta,
\]

where the first term on the right hand side represents the effect on (43) of changes in \(\zeta(t)\) holding relative prices constant and the second term represents the effect on (43) of changes in relative prices holding constant the weights \(\zeta(t)\) and \([1 - \zeta(t)]\). Clearly, \(\Delta^N_c\) represents the production reallocation effect while \(\Delta^N_\theta\) represents the terms of trade effect. The former is negative, as can be seen directly from (43). I show in the Appendix that the latter is positive, despite the fact that now the terms of trade also respond to shifts in the rate of innovation. Moreover, I show in the Appendix that for imitation rates close to zero \(\Delta^N_c + \Delta^N_\theta < 0\). This stems from the fact that for \(m\) close to zero the relative price of Southern products is very low and their weight in the price index is close to zero. In this case a further reduction of the relative price of Southern products has a small effect on the standard of living of Northern residents and the other elements, whose effect is negative, dominate the change in the North's welfare. This proves the following proposition.

\footnote{The sum of these terms is negative if and only if \((1 - \alpha)(L^N/a - \bar{g})/\alpha > \rho\), or \(g(m = 0) > (1 - \alpha)\bar{g}\), where \(g(m = 0) = (1 - \alpha)L^N/a - \alpha \rho\) is the steady-state rate of innovation when the rate of imitation equals zero. Since the steady-state rate of innovation rises with the rate of imitation (Proposition 2), \(g(m = 0) > \bar{g}\) and therefore the second inequality holds.}
Proposition 7: For economies that begin in steady state, a tightening of intellectual property rights brings about welfare losses to the North on account of the reallocation of production and welfare gains on account of changes in the terms of trade. The losses are, however, larger than the gains whenever the rate of imitation is sufficiently small.

Now combine Propositions 6 and 7 in order to obtain the following theorem.

Theorem 2: For economies that begin in steady state with small rates of imitation, tighter intellectual property rights hurt the North.

Theorems 1 and 2 demonstrate an important feature of our world economy: Whenever the rate of imitation is low, there exists no conflict between the two regions with regard to the direction of desired policy changes. On welfare grounds, both gain from somewhat lax intellectual property rights. Small amounts of imitation are Pareto superior to no imitation at all.

4. Foreign Direct Investment

Companies engage in foreign direct investment for a variety of reasons. Sometimes FDI is driven by low labor costs in the host country, sometimes by vertical integration in resource industries, still other times by opportunities to jump tariff walls, or enjoy low corporate tax rates or tax holidays. The decision to go multinational is complicated by institutional factors, such as the difference between the parent country's and the host country's legal systems, language barriers that hamper the interaction of the parent and the subsidiary, uncertainty about exchange rate movements, and the like (see Caves (1982)). In addition to these difficulties less developed countries suffer disadvantages as hosts of FDI because they often provide poor infrastructures, lack minimally required supplies of skilled labor, and suffer from political instability. Consequently Northern-based multinationals invest in developed market economies more than in less developed countries. And their investment in LDCs is biased towards unskilled-labor intensive and resource extraction industries.

It follows that a satisfactory analysis of intellectual property rights that accounts for foreign direct investment requires models that are significantly broader than what I have used so far. Such major extensions are beyond the scope of this paper. On the other hand an approach that presumes that whenever wages are lower in the South innovating companies in the North find it profitable to shift manufacturing facilities to the South greatly oversimplifies the problem at hand. It may even lead to misleading conclusions, and especially so in one-factor, one-sector frameworks. Nevertheless, in order to obtain some feeling for the role that FDI may play in the evaluation of tighter intellectual property rights I provide in this section a simple analysis that builds on this very presumption.
So consider the simple case discussed in Section 2, where innovation takes place at a constant exogenous rate, \( g \), as in equation (1). Unlike in Section 2, however, now suppose that a Northern company can costlessly form a manufacturing subsidiary in the South in order to take advantage of lower labor costs. Assume that the risk of imitation is independent of whether a company goes multinational (a more general approach would allow the rate of imitation to differ between national and multinational corporations, with the risk of imitation perhaps being higher for multinationals). In this case, the values of national and multinational companies are the same. Moreover, the differential equation (2) for the fraction of goods that have not been imitated applies to the current framework, as does its solution (3), except that now we split \( n^N \) into national companies whose measure is \( n^{NN} \) and multinational companies whose measure is \( n^{NM}(n^{NN} + n^{NM} = n^N) \).

Under these circumstances the same equilibrium wage rate

\[
\begin{align*}
(47) \quad w &= w^N = w^S
\end{align*}
\]

prevails everywhere, for otherwise Northern firms would not be active in both regions. All products that have not been imitated, whether they be manufactured by national or by multinational corporations, are equally priced according to (10). All those that have been imitated are priced according to (11). It follows from (7), (10), (11), and (47) that relative per product output flows of national, multinational, and Southern-based companies are given by

\[
\begin{align*}
(48) \quad x^{NM} &= x^{NN}, \\
(49) \quad x^S &= \alpha^{-e} x^{NN},
\end{align*}
\]

where \( x^{NN} \) is the output flow of a Northern-based national firm with monopoly power, \( x^{NM} \) represents the output flow of a Northern-based multinational firm with monopoly power, and \( x^S \) is the output flow of a Southern-based firm.\(^{24}\)

Assuming that all imitated products are produced in the South, the full employment conditions now become (compare to (13))

\[
\begin{align*}
(50a) \quad n^{NN} x^{NN} &= L^N, \\
(50b) \quad n^{NM} x^{NM} + n^S x^S &= L^S.
\end{align*}
\]

\(^{24}\) My use of language presupposes that a product that has been imitated is necessarily manufactured by a Southern-based company. This is not always the case. The original innovator and the imitator face in equilibrium the same marginal manufacturing costs. Therefore if they engage in price competition each one can end up manufacturing the good even if its production takes place in the South. If the Northern firm manufactures the good in the South it means that it is a multinational. Otherwise the good is produced by a Southern-based company. This implies that in addition to the \( n^{NM} \) multinationals there also can exist multinationals with products that have been imitated. The latter type of multinationals is inconsequential for our analysis. I therefore assume for simplicity that a product that has been imitated and is manufactured in the South is supplied by a Southern-based company. Another possibility to which I will come back soon is that a product that has been imitated is nevertheless manufactured in the North. For the time being let us, however, disregard this case.
In the North all employment emanates from Northern-based national companies, while in the South it emanates from both Northern-based multinationals and Southern-based companies. Equations (48)–(50) imply a link between the fraction of multinationals among Northern-based companies whose products have not been imitated, $\mu \equiv n^{NM}/n^N$, and the fraction of products that have not yet been imitated $\zeta$:

\[
\zeta = \frac{\alpha^{-\epsilon} L^N}{L^S + \alpha^{-\epsilon} L^N}
\]

(51) \[\mu = \frac{L^S}{L^S + L^N} - \alpha^{-\epsilon} \frac{1 - \zeta}{\zeta} \frac{L^N}{L^S + L^N}.
\]

This relationship must be satisfied at each point in time. Equation (51) shows that a larger fraction of Northern manufactures is produced abroad the larger is the fraction of products that have not been imitated.

Curve $MM$ in Figure 3 describes the relationship between $\mu$ and $\zeta$ that is embodied in (51). The world economy has to be on this curve at each point in time as long as $\zeta \geq \zeta_c$. The vertical line $\zeta = 0$ describes the rest points of the differential equation (2); the fraction of goods that have not been imitated does not change if and only if it equals $\zeta = g/(g + m)$. For values of $\zeta$ greater than $\zeta_c$ this fraction declines while for values less than $\zeta_c$ it increases. The dynamics of the system are described by the arrowed trajectory; the world economy converges to the stationary point $A$. The critical value $\zeta_c = \alpha^{-\epsilon} L^N/(L^S + \alpha^{-\epsilon} L^N)$ represents the lower bound on the fraction of goods that have not been imitated for which $\mu > 0$. If the initial value of $\zeta$ is smaller than $\zeta_c$ then $\mu = 0$ and the
North manufactures some of the goods that have been imitated. In what follows I confine the discussion to cases with $\xi(0)$ and $\tilde{\xi}$ larger than $\xi_c$.\footnote{The economy has to be either on the horizontal axis between 0 and $\xi_c$, or on $MM$ to the right of $\xi_c$. In the former case the dynamics are along the horizontal axis. Thus, for example, if in Figure 3 the initial value of $\xi$ falls short of $\xi_c$, then $\mu$ remains zero for a while while $\xi$ increases until it reaches $\xi_c$. Afterwards the trajectory proceeds along $MM$ towards point $A$. The meaning of the initial phase is that some of the imitated products are produced in the North, but as the fraction of unimitated products increases fewer of the imitated products are produced in the North. Eventually all imitated products are produced in the South (when $\xi = \xi_c$) and as $\xi$ further increases Northern companies with unimitated products manufacture abroad in order to take advantage of incipient wage differentials.
\footnote{In the absence of foreign direct investment we distinguished between terms of trade effects and the regional allocation of production. With FDI relative prices are constant and we do not have the usual terms of trade effect. On the other hand, the regional allocation of production is not associated in a simple way with high and low priced goods, because due to the multinationals the South supplies both high and low priced goods. What matters for efficiency is of course the association of relative prices with relative marginal costs. In this section an increase in $\xi$ represents an increase in the fraction of highly priced goods without affecting relative marginal costs.}}

Now suppose that the world economy is at point $A$. A tightening of intellectual property rights reduces $m$ and raises $\tilde{\xi}$. The $\xi = 0$ line shifts to the right and $\mu$ and $\xi$ rise gradually along $MM$ from $A$ to the new steady-state point. We conclude that a tightening of IPRs gradually increases the fraction of goods that have not been imitated and the fraction of products with monopoly power that are produced by Northern-based multinationals. What are the welfare implications of these comparative dynamics? In order to answer this question we need to examine each region's flow of utility in the presence of FDI.

In each region the flow of utility equals the logarithm of real spending (see (9)). The price index (12) applies to the current case, except that wage rates are equalized across regions. Taking account of the pricing equations (10) and (11) and the wage structure (47), (12) implies

$$\begin{equation}
\begin{aligned}
P &= n^{1/1-\varepsilon} w \left[ \xi \alpha^{\varepsilon-1} + (1 - \xi) \right]^{1/1-\varepsilon}.
\end{aligned}
\end{equation}
$$

In the South income per capita equals the wage rate $w$. Therefore (9) and (52) imply the flow of utility:

$$\begin{equation}
\begin{aligned}
\log n^S &= \frac{1}{\varepsilon - 1} \log n + \frac{1}{\varepsilon - 1} \log \left[ \xi \alpha^{\varepsilon-1} + (1 - \xi) \right].
\end{aligned}
\end{equation}
$$

This flow declines with the fraction of goods that have not been imitated, because Northern companies with monopoly power price their goods higher than other products. As the fraction of such goods increases real spending declines. Since the number of products available at each point in time does not change with tighter IPRs and the fraction of unimitated products increases, we conclude that, just as in the case with no foreign direct investment, the South loses from tighter intellectual property rights.\footnote{In the absence of foreign direct investment we distinguished between terms of trade effects and the regional allocation of production. With FDI relative prices are constant and we do not have the usual terms of trade effect. On the other hand, the regional allocation of production is not associated in a simple way with high and low priced goods, because due to the multinationals the South supplies both high and low priced goods. What matters for efficiency is of course the association of relative prices with relative marginal costs. In this section an increase in $\xi$ represents an increase in the fraction of highly priced goods without affecting relative marginal costs.} We have therefore proved Proposition 8.
PROPOSITION 8: For economies in steady state with foreign direct investment, tighter intellectual property rights hurt the South.

Next we calculate the flow of utility of a typical Northern resident. The North derives income from wages and profits. Profits are generated from national and multinational companies whose products have not been imitated, and they equal a fraction \((1 - \alpha)\) of revenue. Therefore income equals \(wL^N + (1 - \alpha)p^N(x^{NN} + x^{NN} + x^{NM} + x^{NM})\). Using the pricing equation (10), the wage equalization condition (47), and (48) and (50a), this income level implies that income per capita equals \(\alpha^{-1}w[1 + \mu/(1 - \mu)]\). Together with (9) and (52) this yields the following flow of utility for a representative Northern worker:

\[
\log u^N = \frac{1}{e - 1} \log n + \frac{1}{e - 1} \log \left[ \xi + (1 - \xi)\alpha^{1 - \epsilon} \right] \\
+ \log \left[ 1 + (1 - \alpha)\frac{\mu}{1 - \mu} \right].
\]

Tighter intellectual property rights do not affect the first term on the right hand side, because the number of available products is exogenous at each point in time. The second term declines with tighter IPRs, because the fraction of products that have not been imitated increases at each point in time. An increase in this fraction raises the price index of consumption, as more goods are monopoly priced. The last term rises with tighter IPRs at each point in time, because the fraction of companies with monopoly power that go multinational increases. This raises profit income per capita in the North and thereby the flow of utility. It follows that the North gains from tighter intellectual property rights if the income effect of multinational corporations dominates the consumer price index effect, and it loses otherwise. Are both possibilities viable? The answer is in the affirmative.

First consider the following proposition (see Appendix for proof).

PROPOSITION 9: For economies in steady state with foreign direct investment and small imitation rates, a tightening of intellectual property rights benefits the North.

A comparison of this proposition with Proposition 1 shows that with exogenous innovation whenever the pace of imitation is very slow the North benefits from tighter IPRs in the presence of foreign direct investment and loses in its absence. This difference can be explained as follows. In both cases tighter IPRs increase the fraction of highly priced unimitated products, and thereby reduce welfare. Moreover, with \(m\) close to zero the steady-state fraction of unimitated products is close to one. In this event, the welfare effect of terms of trade changes in the absence of FDI is close to zero and there exists no other avenue for welfare changes. Therefore the North loses. In the presence of FDI the terms of trade do not change at all. But now tighter intellectual property rights
raise Northern income via an expansion in the number of multinationals. As shown in the Appendix, this income effect is large enough to offset the cost of consuming a larger fraction of highly priced products.

We have thus seen that with foreign direct investment and low rates of imitation the North gains from tighter intellectual property rights. Table I reports an example with foreign direct investment in which the North gains from tighter IPRs when the rate of imitation is low (ξ is high) and loses when the rate of imitation is high (ξ is low). Since the South always loses from tighter intellectual property rights, it follows that even with foreign direct investment there exist world economies in which both regions lose from tighter intellectual property rights.27

We can obtain a better understanding of the channels through which intellectual property rights operate on the welfare level of the North by contrasting the case that was discussed in Section 2 with the case discussed in this section, and in particular, by focusing on why the desirability of tighter IPRs depends differently in these cases on the initial rate of imitation. Recall that when FDI is not allowed we were required to evaluate the relative importance of the interregional reallocation of production effect and the terms of trade effect. We saw in Section 2 that in this case the relative importance of the terms of trade effect increases in the initial rate of imitation, and that the terms of trade of the North improve with tighter intellectual property rights. Therefore tighter IPRs become particularly advantageous for the North when the rate of imitation is sufficiently high. On the other hand, when FDI is possible there is no terms of trade effect in our model. In this case we need to consider the relative importance of the interregional reallocation of production effect and the profit income effect. The latter increases with tighter intellectual property rights. Shifting manufacturing from Southern firms to Northern firms increases prices of the respective products by the markup and the associated profit income occurs to the North.

27 In the example in Table I the North loses from tighter IPRs for values of ξ that induce low levels of FDI. In most of the simulations that I have examined, however, the flow of utility in the North rises with tighter IPRs for all values of ξ that ensure a positive level of multinationality. An example of the latter case is a world economy with \( L^N = 1 \); \( L^S = 6 \); and \( \alpha = 0.2 \).
Looking at the impact of these price and profit increases from the point of view of partial equilibrium we would speak of a dead-weight loss. We are analyzing a general equilibrium setting, however. Then, with labor supplied inelastically, distortions need not increase with the extent of monopoly pricing. To see why, take an extreme case in which almost all products are manufactured by the North and priced with a fixed markup above marginal costs. Then there are almost no distortions as long as the labor supply is fixed, because almost all relevant relative prices equal relative costs. Moreover, in this case an extension of monopoly pricing to all products reduces the dead-weight loss, because it brings about equality of relative prices and relative marginal costs for all products. It follows that extending equal monopoly power to all goods brings about welfare gains. Similarly, when monopoly pricing is widespread initially, even if it does not apply to almost all goods, an extension of monopoly power raises welfare. This intuition explains my results: The lower the initial rate of imitation the more products are manufactured by Northern firms, the more goods are priced with a markup, the smaller the “dead-weight loss” (which may even turn into a gain), and the higher the relative importance of the profit income effect. Therefore, when the rate of imitation is sufficiently low the North will gain from tighter intellectual property rights.

5. CONCLUDING REMARKS

Who benefits from tight intellectual property rights in less developed countries? My analysis suggests that if anyone benefits, it is not the South. This answer is robust with respect to all of the variations that I have examined. In the absence of foreign direct investment tighter IPRs move the terms of trade against the South and bring about a reallocation of manufacturing towards higher priced Northern products, which harms the South. If the rate of innovation is responsive to this policy, the rate of innovation rises initially but declines subsequently. The initial acceleration of innovation is, however, insufficient to compensate Southern residents for its eventual decline. Consequently the shift in the time pattern of available products also hurts the South. The last result may not be robust to model specification. But it shows that endogenous innovation does not guarantee benefits to the South from tighter IPRs on account of the R&D investment response. Finally, we have seen in the previous section that the South also loses from tighter intellectual property rights in the presence of foreign direct investment. While Northern multinationals mitigate the effects of tighter IPRs on the South’s terms of trade, they do not eliminate the negative welfare effect of the reallocation of manufacturing that results in higher prices being paid for a larger fraction of products.

Does the North benefit from tighter intellectual property rights in the South? Here the answer is less clear-cut. Unlike in some other models in which there always exists a conflict between the North and the South (see Chin and Grossman (1990) and Deardorff (1992)), we have identified circumstances in which no such conflict exists. In the absence of foreign direct investment both
regions benefit from some relaxation of IPRs when the rate of imitation is low. The benefits of this policy to the South are clear from the previous discussion. The benefits to the North are more subtle. Like the South, the North benefits from having a larger fraction of products priced more cheaply and from having a more desirable time pattern of innovation in cases where inventions are endogenous. When innovation is endogenous, the shift in the North's time pattern of savings and R&D investment cannot generate adverse welfare effects large enough to outweigh the beneficial effects of greater product availability in the long run. Therefore the only potential source of welfare losses for the North is in the detrimental response of its terms of trade to more relaxed IPRs. This effect proves to be very small in my model when the rate of imitation is low. Therefore the North necessarily gains under these circumstances. In more realistic economies the detrimental effect of the terms of trade may not be so small when the rate of imitation is low. Nevertheless, even then there may exist ranges of imitation rates over which some relaxation of intellectual property rights benefits the North.

But absent foreign direct investment, high rates of imitation lead necessarily to a conflict of interest between the North and the South. Then the North prefers tighter intellectual property rights while the South prefers looser rights. The reason for this conflict is that in this case the terms of trade effect becomes very important and the North secures better terms of trade with tighter IPRs. On the other hand, with active foreign direct investment that eliminates terms of trade effects entirely, there always exists a conflict of interest between the North and the South whenever the rate of imitation is low; the North gains from tighter intellectual property rights while the South loses (recall that in the absence of FDI both countries lose from tighter IPRs when the rate of imitation is low). It may also happen that the North always gains from the policy shift, or that it loses just as the South does, when the rate of imitation is high and the fraction of multinational corporations is small.

My discussion suggests that key problems of intellectual property rights, though complex, are amenable to careful analysis. The question: “Are tighter intellectual property rights desirable?” cannot be answered by theoretical arguments alone. The theoretical analysis is most helpful in identifying channels through which regions are affected by such policy changes and circumstances under which the answer goes one way or the other. It also helps to identify the empirical estimates that are needed in order to answer the question. And this is a good example of what much of applied theory is all about. Nevertheless, one has to bear in mind the context of each and every theoretical investigation, and this one is no exception. I have focused on a number of features that I consider to be central to the problem at hand, but these are by no means all the relevant features. For this reason further analysis is called for in order to improve our understanding of this important policy issue. There follow some additional considerations for future research.

First and foremost, I have treated imitation in a rather crude way. In reality imitation is an economic activity much the same as innovation; it requires
resources and it responds to economic incentives. Mansfield, Schwartz, and Wagner (1981) found, for example, that imitation costs are substantial (of the order of 65% of innovation costs). Moreover, in less developed countries imitation is a major effort that involves the development of absorptive capacity for advanced technologies on the one hand and particular efforts to assimilate and adopt foreign technologies on the other. Pack and Westphal (1986) provide a most interesting account of studies that looked into this problem, and they report: "Effort is required in using technological information and accumulating technological knowledge to evaluate and choose technology; to acquire and operate processes and produce products; to manage changes in products, processes, procedures, and organizational arrangements; and to create new technology. This effort takes the form of investments in technological capability, which is the ability to make effective use of technological knowledge" (p. 105). In order to take account of these considerations there is need for considerable extension of the models employed in this paper.

The absorptive capacity of a less developed country depends on its resource base, including the availability of sufficiently skilled labor and a suitable level of organizational know-how. A country's capacity in these areas can be significantly enhanced by the operation of multinational corporations. But this process is not automatic. It requires the transfer of "tacit knowledge" by multinational corporations, which requires in turn: (a) the location in subsidiaries of activities that have the ability to serve this purpose; and (b) the allowance of interactions between representatives of the parent company and local workers in ways that facilitate such transfers. Naturally, the incentives of multinational corporations to activate these channels of technology transfer (or more generally, the transfer of knowledge) are affected by the degree of enforcement of intellectual property rights, and so are the incentives to innovate. My crude theory of foreign direct investment, based as it is on incipient differences in labor costs, is unable to deal with these issues. But as crude as my theory of FDI is, one clear message emerges from the analysis: The extent of foreign direct investment plays a major role in determining the outcome of different degrees of enforcement of intellectual property rights. This message can only be strengthened by adding the above outlined considerations.

One further limitation of my analysis is worthy of special attention. My models have a single input and representative individuals. For this reason they provide insights into conflicts between the North and the South, but not into conflicts within each region. However, in order to better understand the position of each region, we also need to examine intra-regional income distributions. This applies with particular force to the North. For in the North, which is relatively rich in skilled labor, the loss of business that results from lax enforcement of intellectual property rights in the South means primarily the loss of low-skill labor intensive products and therefore the loss of low-skill jobs. This engenders a coincidence of interests between large corporations that lose monopoly rents and unskilled workers who lose jobs and wage income—an
additional complication in the study of desirable limits to intellectual property rights.

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APPENDIX

This appendix contains calculations and proofs that are missing from the main text. The first part provides computations of the characteristic root $-\lambda$ and a component of the characteristic vector $-\Lambda$. It also establishes qualitative relationships between them, as suggested in equation (36). The second part provides proofs of Propositions 5 and 7 while the third part provides a proof of Proposition 9.

Characteristic Root and Vector

The linearized system of (24)--(25) is

\[
\begin{bmatrix}
\xi \\
g
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\xi - \bar{\xi} \\
g - \bar{g}
\end{bmatrix},
\]

where

\[
a_{11} = -(m + g),
\]
\[
a_{12} = \frac{m}{m + g},
\]
\[
a_{21} = \frac{\alpha}{1 - \alpha} (\rho + m + g)^2,
\]
\[
a_{22} = (\rho + m + g) \left( \frac{\alpha}{1 - \alpha} \frac{\bar{g}}{g + m} + 1 \right).
\]

Since the trace of the matrix on the right hand side is positive and its determinant is negative, it has one positive and one negative characteristic root. To ensure long-run convergence we need to choose a zero value for the free coefficient that multiplies the solution part associated with the positive root. Since $g$ is a jump variable while $\xi$ is a state variable, we also have to choose the remaining free coefficient so as to ensure that at time zero the fraction of goods not yet imitated matches the given initial value. This procedure leads to the solution

\[
\begin{align*}
\xi(t) &= \bar{\xi} + [\xi(0) - \bar{\xi}] e^{-\lambda t}, \\
g(t) &= \bar{g} - [\xi(0) - \bar{\xi}] A e^{-\lambda t},
\end{align*}
\]

where $-\lambda$ equals the negative characteristic root (i.e., $\lambda > 0$) and $[1, -A]^T$ represents the characteristic vector associated with the negative characteristic root, with $\Lambda > 0$. By definition $A[1, -\Lambda]^T = -\Lambda[1, -A]^T$, where $A$ is the matrix on the right hand side of (A1). Since the second row of the matrix is positive, we have $\Lambda > 0$. I provide explicit formulas for $\lambda$ and $\Lambda$ below. This solution describes the same directions in the evolution of $\xi$ and $g$ as does Figure 1. The characteristic equation associated with the matrix on the right hand side of (A1) is

\[x^2 - (a_{11} + a_{22})x + (a_{11}a_{22} - a_{12}a_{21}) = 0.\]

The solution for the characteristic roots is

\[
x_{1,2} = \frac{1}{2} [(a_{11} + a_{22}) \pm B],
\]
where

\((A5)\) \[ B = \left[ (a_{11} - a_{22})^2 + 4a_{12}a_{21} \right]^{1/2}. \]

It follows from the definitions of the coefficients \(a_{ij}\) that the root \(x_1 = \frac{1}{2}[(a_{11} + a_{22}) + B]\) is positive while the root \(x_2 = \frac{1}{2}[(a_{11} + a_{22}) - B]\) is negative. Define \(\lambda = -x_2\). Then

\((A6)\) \[ \lambda = \frac{1}{2}[B - (a_{11} + a_{22})] > 0.\]

Now let \([1, v_2]^T\) be the characteristic vector associated with the characteristic root \(x_2\). Then

\((A7)\) \[ a_{11} + a_{12}v_2 = x_2.\]

Define \(\Lambda = -v_2\). Taking account of \(x_2 = -\lambda\) and the definition of \(\Lambda\), equations \((A6)\) and \((A7)\) imply

\((A8)\) \[ \Lambda = \frac{1}{2a_{12}}(B + a_{11} - a_{22}) > 0.\]

**Lemma 1:** \(\Lambda > \alpha \rho\).

**Proof:** From \((A8)\) we obtain \(\Lambda > \alpha \rho\) if and only if \(B^2 > 4\alpha^2 \rho^2 a_{12}^2 + (a_{11} - a_{22})^2 - 4\alpha \rho a_{12}(a_{11} - a_{22})\). Substituting \((A5)\) and the values of the \(a_{ij}\) coefficients from \((A1)\) into this inequality, we obtain \(\Lambda > \alpha \rho\) if and only if \((\rho + m + g)^2 > \Gamma\), where

\((A9)\) \[ \Gamma = \rho \left[ (1 - \alpha)\rho \frac{m}{m + g} + (\rho + m + g) \left( 1 - \frac{\alpha}{m + g} \right) + (1 - \alpha)(m + g) \right]. \]

The right-hand side of \((A9)\) attains a maximum over \(\alpha\) at a negative value of \(\alpha\). Since \(\alpha\) is positive, this implies that \(\Gamma\) does not exceed the value of the right-hand side evaluated at \(\alpha = 0\), namely

\[ \Gamma < \rho^2 + 2\rho(m + g) < (\rho + m + g)^2. \]

It follows from the second inequality that \(\Lambda > \alpha \rho\). \(Q.E.D.\)

**Lemma 2:** \(\Lambda < \alpha(\rho + \lambda)\).

**Proof:** From \((A3)\) and \((A5)\) we obtain that \(\Lambda < \alpha(\rho + \lambda)\) if and only if

\[ B < \frac{2\alpha a_{12}}{1 - \alpha a_{12}}(\rho - a_{11}) + (a_{22} - a_{11}). \]

The last inequality is, however, satisfied if and only if the square of its left-hand side is smaller than the square of its right-hand side. Using \((A5)\) the latter is equivalent to

\[ a_{21} < \frac{a_{21}^2}{(1 - \alpha a_{12})^2}(\rho - a_{11})^2 + \frac{\alpha}{1 - \alpha a_{12}}(\rho - a_{11})(a_{22} - a_{11}). \]

By substituting the expressions for the \(a_{ij}\) coefficients from \((A1)\) into the last inequality one can verify that it holds. It follows that \(\Lambda < \alpha(\rho + \lambda)\). \(Q.E.D.\)

**Terms of Trade**

First we compute \(\Delta_0^S\) and \(\Delta_0^N\). From the definitions of \(\Delta_0^S\) we obtain

\((A10)\) \[ \Delta_0^S = -\frac{\alpha\xi}{\xi + (1 - \xi)\theta} \int_0^\infty e^{-\rho \mu} \frac{d\theta(t)}{d\mu} dt, \]

\((A11)\) \[ \Delta_0^N = \alpha \frac{(1 - \xi)\theta^{a-1}}{\xi + (1 - \xi)\theta^a} \int_0^\infty e^{-\rho \mu} \frac{d\theta(t)}{d\mu} dt, \]

where \(a > 1\).
where $\bar{\theta}$ is the steady state value of $\theta(t)$ (see (40)). From the definitions of $\theta(t)$ in (40) and (30)–(33) we calculate

\begin{equation}
\int_0^\infty e^{-\mu} \frac{d\theta(t)}{d\mu} \, dt = \frac{a \bar{a} \bar{\theta}^2}{D \bar{\xi} L^3} \Delta_\theta,
\end{equation}

where

$$
\Delta_\theta = (1 - \bar{\xi}) \left( -1 + \frac{\lambda}{\alpha(\rho + \lambda)} \right) + \frac{\lambda(\rho + g + m)}{\rho(1 - \alpha)(\rho + \lambda)}.
$$

**Lemma 3:** $\Delta_\theta^L < 0$ and $\Delta_\theta^N > 0$.

**Proof:** From (A10)–(A12) it suffices to prove that $\Delta_\theta > 0$. It follows from the definition of $\Delta_\theta$ and (36) that

\begin{align*}
\Delta_\theta &> -1 + \frac{\lambda}{\alpha(\rho + \lambda)} + \frac{\lambda(\rho + g + m)}{\rho(1 - \alpha)(\rho + \lambda)} \\
&> -1 + \frac{\alpha \rho}{\alpha(\rho + \lambda)} + \frac{\lambda(\rho + g + m)}{\rho(1 - \alpha)(\rho + \lambda)} = \frac{\lambda(m + g + \alpha \rho)}{\rho(1 - \alpha)(\rho + \lambda)} > 0.
\end{align*}

Q.E.D.

**Lemma 4:** For $m$ close to zero $\Delta_\xi^N + \Delta_\theta^N < 0$.

**Proof:** From the definition of $\Delta_\xi^N$ we compute

\begin{equation}
\Delta_\xi^N = -\frac{\bar{g}^a - 1}{\bar{\xi} + (1 - \bar{\xi}) \bar{\theta}^a} \int_0^\infty e^{-\mu} \frac{d\xi(t)}{d\mu} \, dt, \quad \bar{\theta} > 1.
\end{equation}

Together with (30) and (32) this implies

\begin{equation}
\Delta_\xi^N = -\frac{\bar{g}^a - 1}{\bar{\xi} + (1 - \bar{\xi}) \bar{\theta}^a} \frac{\lambda \bar{g}}{D \alpha \rho(\rho + \lambda)}.
\end{equation}

It follows from (A11)–(A13), (27), (28), and (40) that

\begin{equation}
\Delta_\xi^N + \Delta_\theta^N = -\frac{\bar{g} \bar{a}^a}{D \bar{\xi} + (1 - \bar{\xi}) \bar{\theta}^a} \Gamma_e,
\end{equation}

where

\begin{equation}
\Gamma_e = -\frac{1 - (\bar{\theta})^{-a}}{(\rho + \lambda) \alpha \rho} + \frac{\lambda}{\alpha(\rho + \lambda)} \left(-1 + \frac{\lambda}{\alpha(\rho + \lambda)} \right) \frac{\alpha a}{L^N - a \bar{g}} + \frac{\lambda}{\rho \xi(\rho + \lambda)}.
\end{equation}

Next observe that $m \to 0$ implies $\bar{\xi} \to 1$ and $\bar{\theta} \to +\infty$. It is also easy to verify that $\lambda$ and $\Lambda$ remain finite as $m \to 0$. Therefore

\begin{equation}
\Gamma_e \to -\frac{\lambda(1 - \alpha)}{(\rho + \lambda) \alpha \rho} < 0 \quad \text{as} \quad m \to 0.
\end{equation}

It follows from (A14) that $\Delta_\xi^N + \Delta_\theta^N < 0$ for $m$ sufficiently small. Q.E.D.

**Foreign Direct Investment**

In this part I prove Proposition 9, which states that for economies in steady state with FDI and low rates of imitation the North gains from tighter intellectual property rights. In view of the fact that tighter IPRs lead to an increase in $\zeta$ at each point in time and in view of the relationship between $\zeta$ and $\mu$ given in (51), it is sufficient to show that the derivative of the right hand side of
(54) with respect to $\zeta$ is positive for $m \to 0$. Evaluated at $\zeta = \bar{\zeta}$ this derivative equals

$$
\frac{d \log u^N}{d \zeta} = \frac{1 - \alpha}{(1 - \alpha \bar{\mu})(1 - \bar{\mu})} \frac{d \mu}{d \zeta} - \frac{1 - \alpha}{\bar{\zeta} \alpha^{\zeta - 1} + (1 - \bar{\zeta})},
$$

where $\bar{\mu}$ represents the steady state value of $\mu$. Using (51) to calculate $d \mu / d \zeta$ and evaluating the right hand side of (A15) at $m = 0$ (i.e., $\bar{\zeta} = 1$), we obtain

$$
\frac{d \log u^N}{d \zeta} \bigg|_{m=0} = \frac{(1 - \alpha) \alpha^{-\varsigma}}{1 - \alpha \theta^\varsigma} \left[ 1 - (1 - \alpha \theta^\varsigma)(1 - \alpha^{\zeta - 1}) \right] > 0,
$$

where $\theta^\varsigma = L^S / (L^S + L^N)$ represents the share of the South in the world's labor force. Q.E.D.

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